DOCUMENT RESUME

ED 155 025 SE 024 280

TITLE	Mathematics for Junior High School, Pilot Edition, Chapters 8 & 9.
INSTITUTION	Boston Univ., Mass.
SPONS AGENCY	National Science Foundation, Washington, D.C.
PUB DATE	75
GRANT	NSF-SED-74-18105
NOTE	34p.; For related documents, see SE 024 279-283
EDRS PRICE	MF-$0.83 HC-$2.06 Plus Postage.
DESCRIPTORS	Activity Units; *Curriculum; *Instructional Materials; *Junior High Schools; *Mathematics Materials; Percentage; Problem Sets; Sampling; *Secondary School Mathematics; *Textbooks
IDENTIFIERS	*Boston University Mathematics Project; Indirect Measurement

ABSTRACT

This book contains the eighth and ninth chapters of a pilot mathematics sequence for the seventh and eighth grades. The content of the sequence is to serve as a vehicle for the development of relevant computational skills, mathematical reasoning, and geometric perception in three dimensions and is to reflect the application of mathematics to the social and natural sciences. The material is divided into five types of sections: (1) activities; (2) short reading sections; (3) questions; (4) sections for the student with a weaker background; and (5) sections for the strongly motivated student. The material in chapters eight and nine includes indirect measurements and sampling. (MN)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

U.S. DEPARTMENT OF HEALTH, EDUCATION & WELFARE
NATIONAL INSTITUTE OF EDUCATION

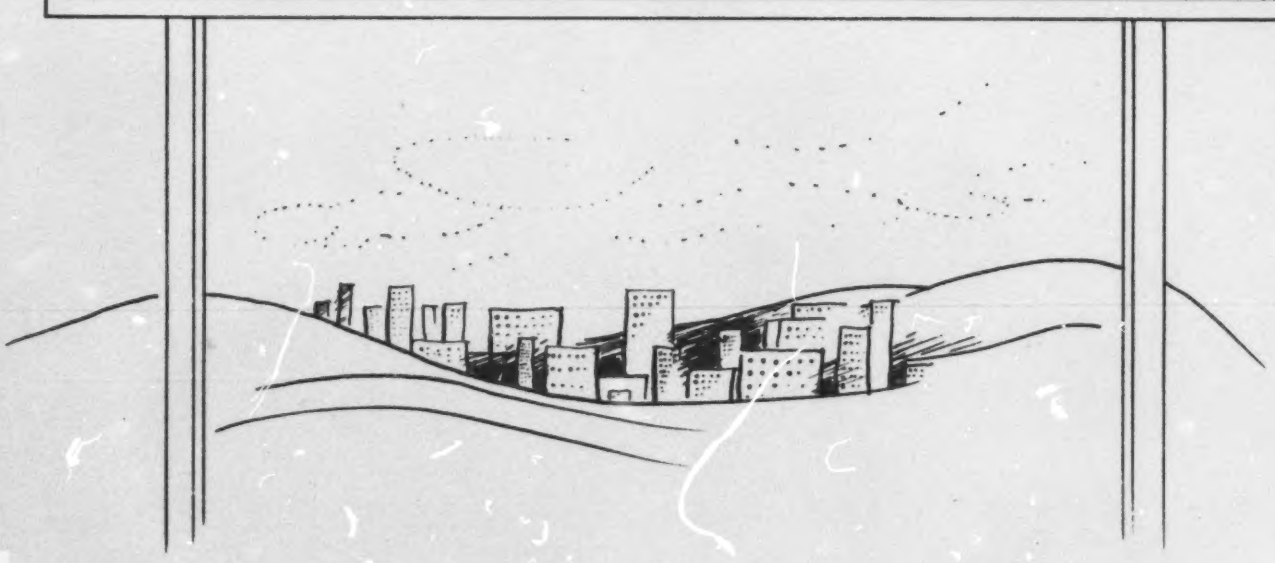
THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL NATIONAL INSTITUTE OF EDUCATION POSITION OR POLICY

ED155025

MATHEMATICS

FOR JUNIOR HIGH SCHOOL

pilot edition — *chapters 8 & 9*



Boston University Mathematics Project

SE 024 280

CONTENTS

CHAPTER 8 INDIRECT MEASUREMENTS

1 Combining Measurement and Calculation
2 Indirect Measurements of Height
3 Your Personal "Range Finder"
4 Other Three-Step Calculations

CHAPTER 9 SAMPLING

1 Coded Messages
2 Frequency of Letters
3 Percent
4 Finding a Percent of the Whole
5 Adding Samples
6 Decoding Using Frequency of Common Letters
7 Using Percent to Find the Whole
8 Using Samples to Count Whales

Copyright © 1975 by The Trustees of Boston University
Printed in the United States of America

Except for rights for materials reserved by others, the Copyright owner hereby grants permission without charge to domestic persons of the United States and Canada for use of this Work in the English language in the United States and Canada after September 1, 1977. For conditions of use and permission to use the Work or any part thereof for foreign publications in other than the English language, apply to the Copyright owner or publisher. Publication pursuant to any permission shall contain an acknowledgement of this copyright and the following statement:

"Some of the materials incorporated in this work were developed with the financial support of National Science Foundation Grant No. SED74-18105. However, any opinions, findings, conclusions, or recommendations expressed herein are those of the authors and do not necessarily reflect the views of the Foundation. The materials under free license constitute an incomplete pilot edition and their publication does not imply approval or disapproval of the original copyright owner. Any revisions of such material are the exclusive responsibility of the revisers."

8. INDIRECT MEASUREMENTS

SECTION 1 COMBINING MEASUREMENT AND CALCULATION

You can use a ruler to measure lengths only up to a few meters. However, by combining measurement and calculation you can extend the use of the ruler to measure much larger distances. You have done this already in Chapter 5. There you measured a distance on a map and then calculated the real distance by multiplying the measured one by a scaling factor. You combined measurement and calculation by using the equation:

new length = (scaling factor) × (original length)

1. Turn back to Figure 2 of Chapter 5. What is the length and the width of the ice cream shop on Maple Avenue?

2. A triangle is enlarged 3.5 times. One of its sides is 6.3 m long. How long is the corresponding side of the enlarged triangle?

3. When viewed through a microscope a hair appears to be 2 cm thick. The microscope has a magnification of 200. How thick is the actual hair?

In many cases one length can be measured but the scaling factor is not given directly and must be found from other information. An example of such a situation is illustrated in Figure 1: The lower triangle is an enlargement of the upper one. The base of the upper triangle is 5.0 cm long. How long is the base of the lower triangle?

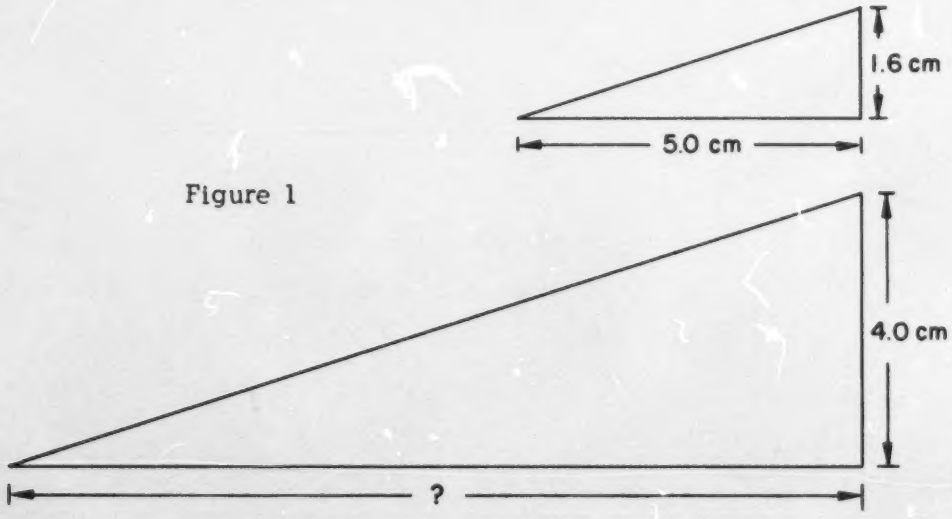


Figure 1

The heights of both triangles are given. The ratio of the new height to the original height gives us the missing scaling factor. Setting up the ratio, as we did in Section 5 of Chapter 6, we get:

$$\text{scaling factor} = \frac{\text{new height}}{\text{original height}} = \frac{4.0 \text{ cm}}{1.6 \text{ cm}}$$

Now in three steps we can answer the question "How long is the base of the lower triangle?" First, we write down the relation between the two bases <u>as if we knew</u> the scaling factor:

longer base = (scaling factor) × 5.0 cm

Next, we calculate the scaling factor from the ratio of the heights:

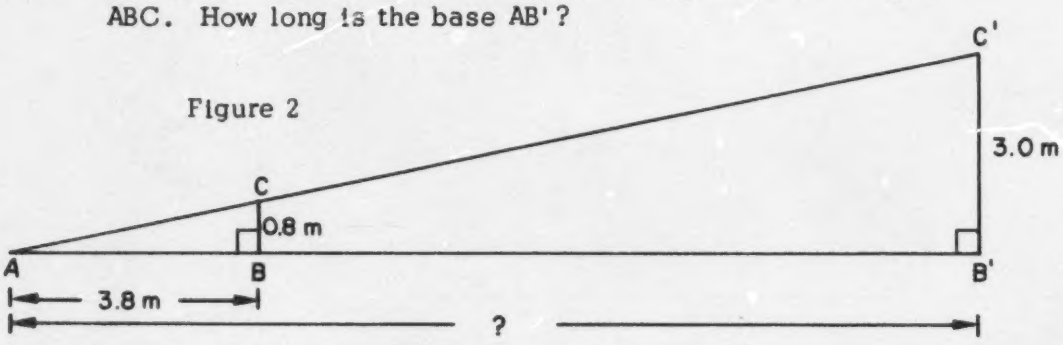
$$\text{scaling factor} = \frac{4.0 \text{ cm}}{1.6 \text{ cm}} = 2.5$$

Finally, we use the value of the scaling factor we have just calculated to find the length of the longer base:

longer base = 2.5 × 5.0 cm = 12.5 cm

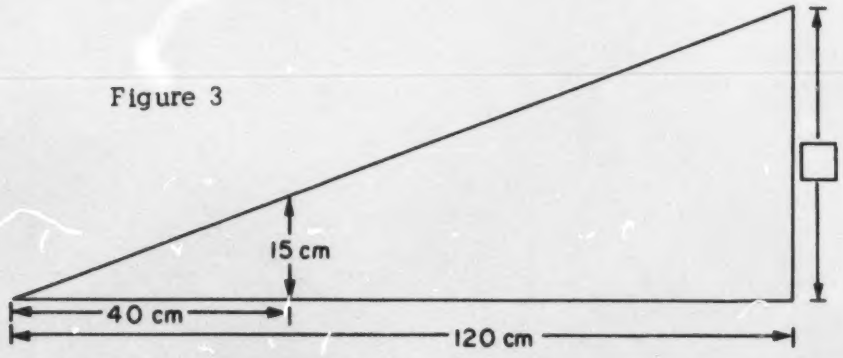
4. In Figure 2 the triangle AB'C' is an enlargement of triangle ABC. How long is the base AB'?

Figure 2



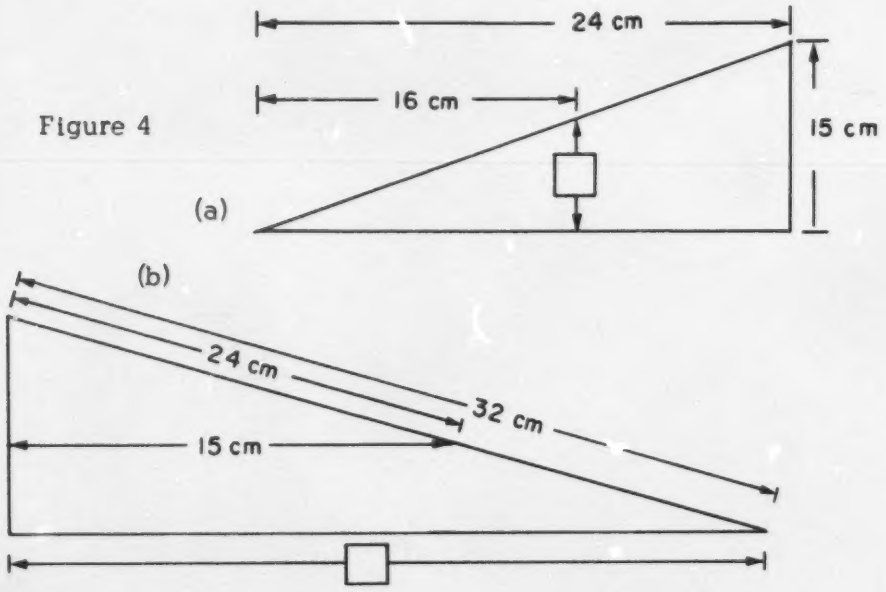
5. Figure 3 shows a pair of overlapping similar triangles. Find the missing length. (The drawing is not done to scale.)

Figure 3



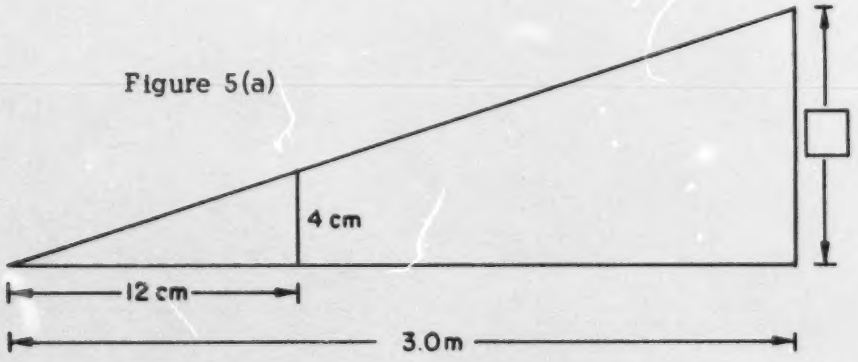
6. Find the missing lengths in Figure 4(a) and (b). (The drawings are not done to scale.)

Figure 4



7. Find the missing lengths in Figure 5(a) and (b). Remember that a scaling factor is a ratio. Therefore, the numbers you divide must represent lengths measured in the same units.

Figure 5(a)



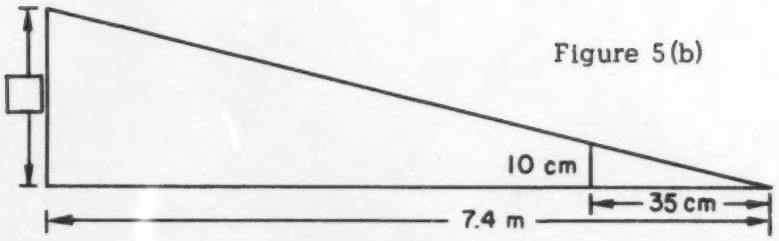


Figure 5(b)

8. A snapshot of a friend shows him holding a large fish he caught on his vacation. He has forgotten how big the fish was. Your friend is 5 ft. 5 in. tall (1.65 m). Using the sketch of the snapshot shown in Figure 6, find the length of the fish.

Figure 6

9. On his way home from school, a boy paces off the length of the shadow cast by a roadside mailbox. The mailbox is 1.30 m high; the shadow is about 3.5 paces long. He also paces off the shadow cast by a nearby barn; it is 45 paces long. Draw the two similar triangles that describe the situation. What is the height, in meters, of the barn?

10. A map of a route connecting Sioux City and Kansas City is shown in Figure 7. The scaling factor is not given. Suppose you know that the distance between Sioux City and Omaha is 150 km.

(a) What is the distance from Omaha to Kansas City?

(b) What is the distance from Sioux City to Kansas City?

Figure 7

SECTION 2 INDIRECT MEASUREMENTS OF HEIGHT

Figures 8 and 9 illustrate a method of finding heights by a combination of measurements and calculation. In order to find the height of the wall in Figure 8, we need to know the length BB' and the scaling factor. We can read the length BB' directly on the meter stick. Because we do not know any pair of other corresponding sides of the triangles in Figure 8, it appears that we cannot compute the scaling factor for the equation

$$\square = (\text{scaling factor}) \times (\text{length BB'})$$

Figure 9 shows the same triangles as those in Figure 8 but with line segment AM' drawn at eye level, parallel to the floor in Figure 8. Lengths AM and AM' are corresponding distances on the original triangles, and so their ratio will give us the scaling factor. Furthermore, you can see that DC = AM' and DE = AM; therefore, you can find the scaling factor from the measurements of DE and DC.

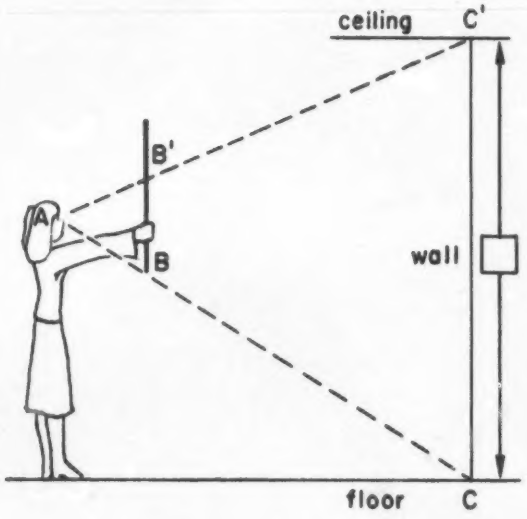


Figure 8

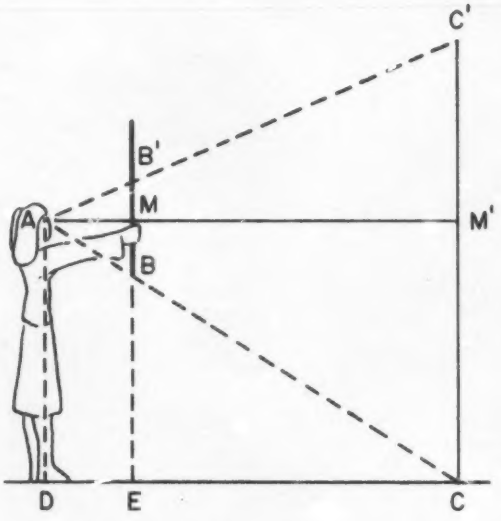


Figure 9

We find the scaling factor from the equation

$$\text{scaling factor} = \frac{DC}{DE}$$

and we use this value in our first equation.

Split up into five groups. Each group chooses some object in the room that has a height or a width that can be measured both indirectly and directly. Each group tells the other groups what it is they want measured.

Members from each group then measure, by the indirect method we have described, the objects chosen by each of the other four groups.

After all the objects have been measured indirectly, someone from each group measures directly the object his or her group chose.

Each group now calculates the sum of the errors of the indirect measurements. The group with the smallest total difference wins.

SECTION 3 YOUR PERSONAL "RANGE FINDER"

A range finder is a device for measuring distances. Your arm, hand, and a distant familiar object make up your personal range finder. It is easy to use when you want to find approximate distances outdoors.

Figure 10 shows you how to use your range finder. If you know the lengths of your arm and thumb and the height of the familiar object, you can calculate the distance.

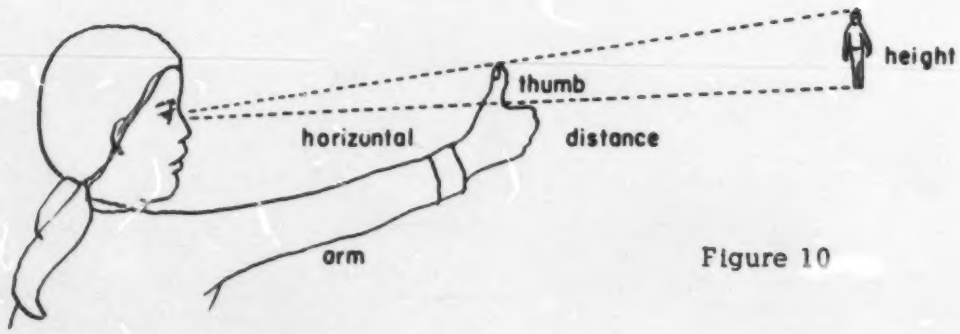


Figure 10

Not all distant objects will appear to be one thumb tall. Therefore, you will need to know the measurements of different parts of your hand. You will also need to know the approximate lengths of some familiar objects.

On an index card, copy the list shown in Figure 11 and fill in the missing lengths. Bring the card with you when you try out your range finder.

PERSONAL MEASUREMENTS	FAMILIAR LENGTHS
Arm length	Height of person
Thumb length	Length of car
Hand width	Height of one story
Little finger width	of a building
	Height of telephone
	pole

Figure 11

In order to use your range finder, you match up a distant height or width with a part of your hand; for example, with the width of a finger or with one-half the length of your thumb. Next you calculate the distance. Since the result will be only an approximation of the actual distance, you may round off to make the calculations simpler.

Try the following with your range finder:

- About how far away must your friend stand from you so that your friend appears to be just as tall as your thumb?
- About how far away are you from a distant car?
  . . . a distant building?

SECTION 4 OTHER THREE-STEP CALCULATIONS

All your calculations of distances in this chapter have one thing in common: They involve three steps. You start with the equation that helps you find the answer although one of the needed quantities is not known. You find the missing value of this quantity from other information given in the question. Finally you use this value in the main equation to calculate the answer.

This method of solving problems is not limited to calculating lengths. It has many other applications. Here are two examples:

Example 1

If one square meter of tiles costs \$2.00, how much will it cost to tile a bathroom that is 2 m by 3 m?

To answer this question, we first write:

$$\text{cost} = \text{number of square meters} \times 2.00\$$$

Next, we find the missing number:

$$\text{number of square meters} = 2 \times 3 = 6$$

Finally, we substitute the 6 in the first equation:

$$\text{cost} = 6 \times 2.00\$ = 12.00\$$$

Example 2

Suppose you have \$5.00 and you want to buy three pens for 40¢ each. How much money will you have left? To find out you subtract what you will spend from what you have:

$$\text{money left} = \$5.00 - \text{cost of pens}$$

Next, you find how much you will spend:

$$\text{cost of pens} = 3 \times 40¢ = 1.20\$$$

Now you use this result in the first equation:

$$\text{money left} = 5.00\$ - 1.20\$ = 3.80\$$$

In answering Questions 11 - 16 write your answer first using an unknown number, show how you will find its value, then use it to compute your answer.

11. One square meter of tiles costs \$2.00. How much does it cost to tile a room 4 m by 5 m?

12. The cost of a field trip is \$2.75 for each student.

    (a) What is the total cost of the trip for a school of six classes with 30 students in each class?

    (b) What is the total cost of the trip for a school of two classes if one class has 35 students and the other class has 32 students?

13. During one afternoon 53 persons purchased tickets to a show and 17 returned their tickets and got a refund. Each ticket cost \$1.75. How much money did the cashier turn in that afternoon?

14. A worker is paid \$170.00 for 40 hours of work. One week he worked only 32 hours. How much was he paid?

15. A length of one meter of chain link fence costs $6.50. What does it cost to fence off a rectangular yard 20 m by 30 m?

16. In some states there is a sales tax of 5¢ on each dollar of all purchases. What would be the total tax on three items costing $6.19, $2.75, and $4.85 respectively?

9. SAMPLING

SECTION 1 CODED MESSAGES

How would you send your friend a written message so that only he or she could read it? This question was raised by people many centuries ago. The Roman general Julius Caesar solved this question by using a code. His code consisted of shifting each letter in the alphabet three places ahead: A became D, B became E, and so on. The complete code used by Julius Caesar is shown in Figure 1. (The Romans at that time had not yet used the letters Y and Z, but we have added them.)

A code in which each letter is shifted a fixed number of places is called a translation code.

Figure 1

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C

To write a message in Caesar's code you first write it in plain English. Then you replace each letter by the letter below it in the second line of Figure 1. To give you a feeling for the coding, write the following message in Caesar's code (use capital letters for clarity):

> From the lamp post go two thousand yards north and forty yards west. Under the big rock is a brown boot.

The coded message makes no sense for someone who does not know the code. Yet in its coded form some clues may be found. For example, the letters A and I in the English language are the only single letters that make words. Therefore, if the letters are correctly grouped, then the single letter D (in the coded message) must stand for A or I.

To make it harder to break a code you can write the coded message in groups of five letters each. Do this with the message you have just coded.

SECTION 2 FREQUENCY OF LETTERS

Now that you have written a message in code try to break the code of the following message:

FTUEO TMBFQ DIUXX ETAIK AGEQH QDMXP
URRQD QZFIM KEFAO APQMZ PPQOA PQYQE
EMSQE UZADP QDFAN DQMWM HQDKP URRUO
GXFOA PQMOD KBFAS DMBTQ DYGEF RUDEF
GZPQD EFMZP FTQEF MFUEF UOMXY QFTAP
EFTMF KAGIU XXXQM DZUZN GYB

What clues can you use? Do some letters appear more often than others? To find out, write the alphabet on a sheet of paper. Now go through the coded message and put a tally mark (see Figure 2) beside the letters on your paper for each letter that appears in the message.

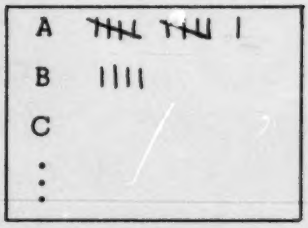


Figure 2

Which letter appears most frequently? Which is in second place? Third place?

It is likely that the most frequent letter in the coded passage corresponds to the most frequent letter in English. Thus your tally may help you in breaking the code only if you know which letters appear most often in regular English.

Make a tally of the letters in the passage in Section 1 and a separate tally for a passage from any book written in English.

Which letter appeared most frequently in the passage of your choice and in the passage from Section 1?

Suppose the code used is a translation code. Which letter appeared most often in your classmates' passages? Can you be sure, on the basis of any one passage, what the code is?

SECTION 3 PERCENT

The number of E's in two passages are counted. The first passage has eight E's out of a total of 50 letters. The second passage has four E's out of a total of 20 letters. In which passage are the E's more frequent?

Because the passages are of different lengths we cannot directly compare the number of E's in the two passages. We must somehow take the size of the passage into consideration. One way to do this is to calculate for each passage the ratio of the number of E's to the total number of letters.

In the first passage we have eight E's out of 50 letters. Therefore the ratio of the number of E's to the number of letters is $\frac{8}{50}$. Similarly, in the second passage the ratio of the number of E's to the number of letters is $\frac{4}{20}$. Which of the two ratios is greater?

When we compare ratios we often change them to equivalent ratios in hundredths.

Since $\frac{8}{50} = 0.16$ and $\frac{4}{20} = 0.20$ we see that the ratio of E's to the number of letters is greater in the second passage.

We have a special name for ratios expressed in hundredths. The number of hundredths in such a ratio is called the number of percent. The word "percent" comes from the Latin per centum, which literally means "per hundred." Often the percent symbol % is used to show the number of hundredths. We can write "16 hundredths of the letters are E's" as "16 percent of the letters are E's" or as "16% of the letters are E's."

Let us look at another example. A school team won 12 of the 20 games it played. What percent of games played did the team win?

The ratio of games won to games played is $\frac{12}{20}$. In hun-

dredths this is 0.60. Therefore the team won 60 percent of the games it played. The number of percent is often called the <u>percentage</u>.

1. Copy and complete these equations:

   (a) $\frac{9}{10} = \frac{90}{100} =$ %

   (b) $\frac{1}{2} = \frac{\quad}{100} =$ %

   (c) $\frac{1}{4} = \frac{\quad}{100} =$ %

   (d) $\frac{2}{5} = \frac{\quad}{100} =$ %

   (e) $\frac{1}{25} = \frac{\quad}{100} =$ %

2. Write these fractions as percents:

   (a) $\frac{3}{4}$

   (b) $\frac{7}{10}$

   (c) $\frac{1}{5}$

   (d) $\frac{19}{100}$

   (e) $\frac{49}{50}$

   (f) $\frac{11}{20}$

3. Copy and complete:

(a) $0.25 = \frac{25}{100} =$ %

(b) $0.45 = \frac{\quad}{100} =$ %

(c) $0.3 = \frac{\quad}{100} =$ %

(d) $0.5 = \frac{\quad}{100} =$ %

(e) $0.75 = \frac{\quad}{100} =$ %

(f) $0.8 = \frac{\quad}{100} =$ %

4. On a Friday only 20 of the 25 students were present. What percent of the class was present? What percent was absent?

5. On a history examination there was a total of 50 points. What percent scores did these students get, if their point totals were

Alice	30
Guillermo	38
Mario	41
Patricia	35

6. Yaz took three spelling tests. On the first test, Yaz spelled 8 out of 10 words correctly. On the second, he spelled 18 out of 20 correctly. On the third, he got 18 out of 25 words right.

(a) On which test did Yaz get the best (percent) score?

(b) On which did he get the worst score?

7. In a basketball game a player attempted 20 set shots and made 12. Another player made 9 out of 16 set shot attempts. Which player made a greater percentage of attempted set shots?

You can change a fraction to a percent in several ways. One way is to find an equal fraction with 100 in the denominator. This works because the numbers of hundredths is the number of percent. You probably used this way in answering Questions 1 - 7.

Another way is to rewrite the fraction as a quotient and divide. Again the number of hundredths gives you the percentage. For example,

$$\frac{26}{40} = 26 \div 40 = 0.65$$

that is,

$$\frac{26}{40} = 65\%$$

This method also works when the answer does not come out as a whole number of hundredths. For example: What is $\frac{1}{8}$ as a percent?

$$\frac{1}{8} = 1 \div 8 = 0.125$$

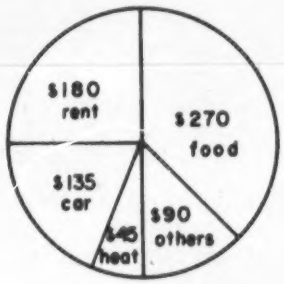
Renaming 0.125 in hundredths we read 12.5 hundredths or 12.5 percent.

8. Change the following fractions to percents:

(a) $\frac{3}{8}$ (b) $\frac{5}{8}$ (c) $\frac{9}{40}$ (d) $\frac{31}{40}$ (e) $\frac{100}{125}$

9. The total amount of money Jeanette's family spends each month is shown by the pie chart in Figure 3. What percentage of the total amount was spent on each item?

Figure 3



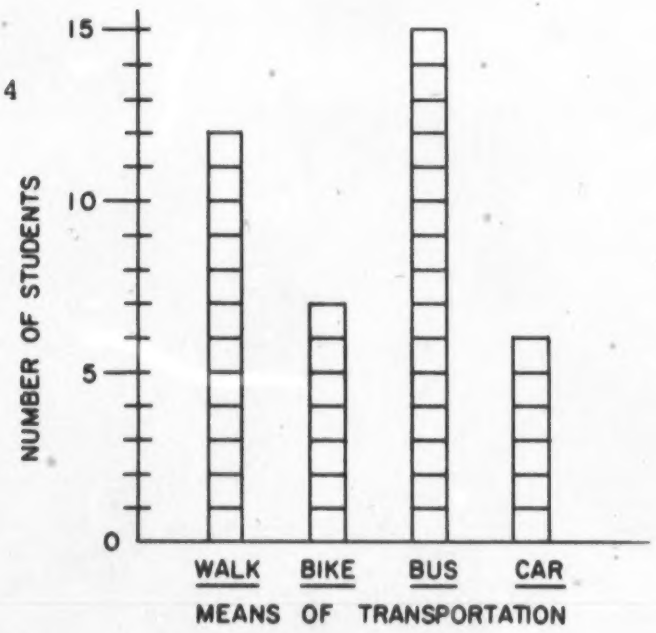
10. The bar graph in Figure 4 shows the number of students who come to class by different means of transportation.

(a) What percentage of the students walk to school?

(b) What percentage come by either bus or car?

(c) What percentage come by bike?

Figure 4



11. What percentage of each circle is shaded?

Figure 5

12. One-quarter of a class has black hair. Three-eighths of the same class has brown hair. What percent of the class has
    (a) black hair?
    (b) brown hair?
    (c) either black or brown hair?

Some fractions do not have exact decimal or percent equivalents. For example, when we try to find the percent value of $\frac{1}{3}$ by dividing, we find

$$\frac{1}{3} = 0.33 \text{ to the nearest hundredth}$$

or

$$\frac{1}{3} = 0.333 \text{ to the nearest thousandth}$$

In cases in which a fraction does not have an exact percent equivalent, we first decide how accurate we want the percent to be and then divide. Thus, $\frac{1}{3} = 33\%$ to the nearest percent, or $\frac{1}{3} = 33.3\%$ to the nearest tenth of one percent.

13. Knoth High School's football team won 10 out of 18 games, and Jordan High School's team won 8 out of 15 games. Which team had the greater percentage of wins?

14. What percentage of each clock face is shaded?

Figure 6

15. Which is larger, $\frac{21}{24}$ or $\frac{22}{25}$? Change to percents to find out.

Your teacher will supply you with a page showing a "scrambled square." Cut out the 16 small squares and reassemble them into another four-by-four square so that all the neighboring numbers match. For example, 15 percent matches $\frac{3}{20}$ because 15 percent equals $\frac{3}{20}$. (The four squares that have only two numbers are corner squares.)

SECTION 4 FINDING A PERCENT OF THE WHOLE

There are 200 letters, and 14 percent of them are E's. How many E's are there? In other words, what is 14% of 200? Since 14% = 0.14, the answer is

$$0.14 \times 200 = 28$$

Therefore 28 of the 200 letters are E's.

16. George got a score of 60 percent on a test of 20 questions. How many questions did he get right?

17. Find

    (a) 15 percent of 200 letters

    (b) 5 percent of 40 students

    (c) 7 percent of a $2.00 item

    (d) 50 percent of 360 degrees

    (e) 20 percent of $10.00

18. If your allowance for one week is $5.00, and you can save 20 percent, how many dollars can you save?

19. In a school of 1250, 28 percent have had the flu this year. How many students had the flu?

20. A bank pays $7\frac{1}{2}$ percent interest each year. If you have $50.00 in a savings account in that bank, how much interest will you get after one year?

21. About 30 percent of your day is spent with your eyes closed. For how many hours do you have your eyes closed each day?

22. Usually I type at 60 words per minute, but now I am out of practice. Now I type at 90 percent of my usual speed. What is my present speed?

23. A local store is selling bicycles at a 20 percent discount. If one kind usually sells for $45.00,

    (a) how many dollars are taken off the price?

    (b) what is the reduced price?

24. A store is selling radios at a reduced price. One radio is reduced from $40.00 to $34.00. What percent of the larger price is the reduced price?

25. If you buy a lunch for $1.60 and a 5 percent tax is added to the cost of your lunch

(a) what is the meal tax?

(b) how much is the total bill?

SECTION 5 ADDING SAMPLES

What are the percentages of the five most frequent letters in the English language? Use the samples you collected in Section 2 of this chapter to find the answers.

In your sample of the letters what is the percentage of the E's? Compare your answer with that of your classmates.

What is the largest percentage of E's your class found? What is the smallest percentage of E's?

Combine your sample with one of your classmates'. How many E's do you have together? How many letters do you have together? What is the percentage of E's?

Compare your percentage with those of your classmates. What is the largest percentage of E's? What is the smallest? How does this compare with what you found with the smaller samples?

What happens to the percentage of E's as the samples get larger? Which samples better describe the percentage of E's in the English language?

Form two groups and add the tallies of E's. What is the percentage of E's in each group?

Make similar tallies for the next four most frequent letters. What are they?

26. Are these valid statements? Why or why not?

(a) Eight of the 10 people surveyed favored basketball. Therefore basketball is the most popular sport.

(b) Of the dentists surveyed, 75 percent agreed that sugarless gum prevents cavities. One dentist did not agree.

(c) The book I am reading is filled with action. I can tell from the first two paragraphs.

27. Baseball averages are given by the number of hits the player would have if he were at bat 1000 times.

(a) Randy Smith is trying out for the Phillies at spring training. He has 6 at-bats and 3 hits. What is his batting average?

(b) Lou Foster is also trying out for the Phillies. Last year he played for the Phillies and had 71 hits with 242 at-bats. What was his average last year?

(c) Lou Foster has not batted in spring training yet. If you were the manager of the Phillies, which player would you regard as the better batter?

(d) Suppose Randy strikes out the next four times at bat, what will his average be?

(e) If Lou Foster had batted four more times last year and struck out each time, what would his average have been?

SECTION 6 DECODING USING FREQUENCY OF COMMON LETTERS

In the code used in Section 2, decoding one letter was enough to break the code. Decoding one letter, however, does not break a code like the one in Figure 7.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
J	Y	E	T	D	P	U	F	R	I	B	N	G	H	X	A	Z	V	Q	W	L	O	C	S	K	M

Figure 7

The code shown in Figure 7 is called a general substitution code. General substitution codes are hard to break. So, we will use correct word lengths in the coded messages.

There are two types of clues you can use to break a substitution code. One type of clue is the frequency of letters. By comparing percentages of the coded message with percentages in the English language, we get clues to what the letters might be. The other type of clue is based on things we know about the English language. For example, a one-letter word in the code must be a or I. Also the most common three-letter word is the.

Decode the message in Figure 8.

JR RLK KQC MZ RLK XDMOB RLKSK TA
J ARMSK T JN QMR AWSK RLJR RLKU
JSK MHKQ XKOJWAK RMCJU TA AWQCJU
TZ UMW OJQ OMNK XJOB RMNMSSMF RLK
ARMSK FTDD XK MHKQ RLKQ

Figure 8

SECTION 7 USING PERCENT TO FIND THE WHOLE

Suppose someone tells you that a rather long paragraph has 42 E's. Can you use this information to calculate the total number of letters in the paragraph?

Usually, 13 percent of the letters in a long passage are E's. Since we have 42 E's, we know that 0.13 of the whole passage has 42 letters. We know 0.13 of the passage, and we want to know the whole passage. This is like knowing something about many and wanting to find something about one. In Section 2 of Chapter 6, we learned that to go from many to one we divide:

number of letters in the whole passage = 42 letters ÷ 0.13
= 323 letters

Of course, the percentage of E's varies from passage to passage. Therefore, we can conclude that this particular passage has about 320 letters.

Pick a passage of English that is about 15 lines long. Count the number of E's and estimate the total number of letters. Check your estimate by counting.

Estimate the number of letters in another passage by counting T's or A's.

28. (a) 20 percent of □ = 45
(b) 6.5 percent of □ = 36.4
(c) 48 percent of □ = 24

29. (a) 20 is 20 percent of what number?

(b) 4 is 25 percent of what number?

(c) 21 is 70 percent of what number?

(d) 8 is 5 percent of what number?

(e) 8 is 1 percent of what number?

30. When necessary, write the answers to the nearest tenth.

(a) $\frac{3}{10} = \square$ percent

(b) 17 is what percent of 85?

(c) what is 3 percent of 230?

(d) 90 percent of what number is 135?

(e) 16.5 percent of 36 = $\square$

31. (a) Forty of the 90 students are boys. To the nearest percent, what percent of the students are girls?

(b) I got 54 questions correct on a test and received 72 percent. How many questions were on the quiz?

(c) Tom has a 280 batting average. He has 125 at-bats; how many hits has he accumulated?

(d) There are 34 boys in our class. They represent 40 percent of the entire class. How many girls are there?

The answers to Questions 32 - 35 involve several steps. As in Section 4 of Chapter 8, you may find it helpful to start with the main equation. Then find values of unknown quantities and use them in the main equation.

32. Last year the enrollment at Day Junior High School was 400. This year the enrollment is up 15 percent. What is this year's enrollment?

33. Every Wednesday the Cape Summer Theater presents a performance in the afternoon and another in the evening. The theater seats 360 people. One Wednesday 65 percent of the seats were filled during the afternoon performance and 85 percent were filled that evening. How many people attended the Cape Summer Theater that day?

34. Andy has saved 25 percent of his earnings. His brother saved twice as much. Andy earned $56.00. Together, how much have the boys saved?

35. In New York City if you buy a pair of slacks that costs $15.00, a sales tax of $1.20 is added to the price. How much tax would there be on a $43.00 sport coat?

SECTION 8 USING SAMPLES TO COUNT WHALES

In the twentieth century, whaling equipment improved greatly. This allowed many more whales to be caught. The International Whaling Commission was set up in 1947 to study the situation, to establish size regulations and quotas, and, if necessary, to ban the hunting of types of whales that were in danger of extinction. Of course, the people who caught whales and the people who used different parts of the whale were opposed to such restrictions.

In order to make fair decisions, the Commission needed to answer the question "How many whales of a particular kind are there?"

This is a hard question. We are going to look at how sampling can provide at least a good estimate of the answer. Instead

of estimating the number of whales, we are going to estimate the number of beans in a jar.

First, mark 200 of the beans, put them back with the others and shake the jar well. Now, without looking, pull out a handful. What percent of our sample is marked?

Assume that the percentage marked in the whole jar is about the same as in your sample. With this assumption calculate the number of beans in the jar using the method you learned in Section 7. Remember that 200 beans are marked.

Can you imagine how this method was used to estimate the number of whales?